Noise Reduction of Electrocardiogram (ECG) Signals Using Singular Value Decomposition in Cardiac Monitoring Systems

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***Abstract*— Electrocardiogram (ECG) signals are a vital tool in clinical practice for monitoring heart activity, yet they are frequently contaminated by various types of noise that can degrade signal quality and lead to misdiagnosis. This study investigates the application of Singular Value Decomposition (SVD) as a robust mathematical approach to enhance ECG signals. By leveraging the ability of SVD to separate meaningful cardiac data from random disturbances based on energy significance, the proposed method effectively suppresses baseline wander and other artifacts. Experimental results demonstrate that this technique successfully produces a cleaner, more stable signal while carefully preserving essential physiological features. Ultimately, SVD-based denoising provides a balanced solution for improving the reliability of cardiac monitoring and automated diagnostic systems.**

***Keywords*— signal enhancement, denoising, Singular Value Decomposition, electrocardiogram**

I. Introduction

Electrocardiogram (ECG) signals are widely used in clinical practice to monitor cardiac activity and diagnose heart conditions, including arrythmias and myocardial infarctions. An ECG waveform is supposed to reflect the electrical activity of the heart. But in real-world measurements it is often contaminated by various type of noise, such as baseline wander, muscle artifacts, and electromagnetic interference, which can significantly degrade signal quality and complicate accurate feature interpretation by medical professionals. The presence of noise not only obscures important ECG morphological features, but also poses challenges to automatic analysis that depend on clear signal characteristics for reliable diagnosis.

Traditional methods for ECG denoising, such as wavelet transform and adaptive filtering, have been applied extensively to mitigate noise effects. Wavelet-based algorithms exploit multi-resolution analysis to suppress unwanted components across frequency bands, demonstrating measurable improvements in signal-to-noise ratio under certain noise conditions. However, these approaches can sometimes distort the diagnostic morphology of the ECG signal or require careful tuning of decomposition levels and thresholding criteria to be effective.

Singular Value Decomposition (SVD) has emerged as a powerful linear algebra tool for signal enhancement and noise reduction due to its ability to decompose a signal into a set of orthogonal components ordered by significance. SVD-based filtering has been studied as a subspace technique to model ECG as a structured signal corrupted by additive noise, whereby the lower singular values, which correspond to noise-dominated components, can be truncated to reconstruct a cleaner signal. Studies focusing on transform-domain SVD filters indicate that such approaches can effectively reduce muscle noise exercise ECG recordings with favorable performance relative to traditional filters.

Given the critical importance of accurate ECG interpretation in cardiac monitoring systems, noise reduction remain a central focus of biomedical signal processing research. This study investigates the application of Singular Value Decomposition for ECG denoising, specifically exploring truncated SVD techniques to enhance ECG signal quality while retaining clinically relevant waveform features, thereby contributing to more reliable cardiac diagnosis and monitoring.

II. Literature Review

A. Electrocardiogram (ECG)

The electrocardiogram (ECG) is a non-invasive diagnostic tool used to record the electrical activity of the heart over time. It captures voltage differences generated by depolarization and repolarization processes of cardiac muscle cells, providing critical information about heart rhythm, conduction pathways, and myocardial health. Since its introduction by Willem Einthoven in the early 20th century, ECG has become a cornerstone in clinical cardiology and biomedical signal analysis.

ECG signals are typically acquired by placing electrodes on the surface of the body according to standardized lead configurations, such as the 12-lead ECG system. These leads provide different spatial perspectives of cardiac electrical activity, enabling comprehensive assessment of cardiac function.

The ECG waveform reflects the electrical conduction system of the heart, which includes the sinoatrial (SA) node, atrioventricular (AV) node, bundle of His, and Purkinje fibers. The propagation of electrical impulses through these structures results in characteristic waveform components:

P wave	: atrial depolarization
QRS complex	: rapid ventricular depolarization
T wave	: ventricular repolarization

These components are directly linked to specific physiological events and are essential for identifying normal and pathological cardiac conditions. Abnormalities in amplitude, duration, or morphology of these waves may indicate arrhythmias, ischemia, myocardial infarction, or conduction block disorders.

From a signal processing perspective, an ECG is a one-dimensional time-series signal representing voltage variations over time. Modern ECG systems digitize the signal using analog-to-digital converters (ADCs), producing discrete samples at a fixed sampling rate. Typical sampling frequencies range from 250 Hz to 500 Hz, which are sufficient to capture clinically relevant cardiac dynamics. Mathematically, a digitized ECG can be represented as:

$$x[n] = x(nT_s), \quad n = 0,1,2, \ldots$$

where T_s is the sampling interval and x[n] denotes the sampled ECG amplitude at time index n. This discrete representation enables computational analysis, storage, transmission, and further mathematical modeling of ECG signals.

B. Noise

In signal processing, noise refers to any unwanted disturbance that contaminates a signal and degrades its quality or interpretability. Noise can arise from various sources, including sensor imperfections, environmental interference, transmission errors, and quantization effects. In mathematical terms, a measured signal y(t) is often modeled as:

$$y(t) = s(t) + n(t)$$

where s(t) is the true signal and n(t) represents noise. Noise negatively impacts tasks such as feature extraction, classification, and parameter estimation. Consequently, understanding noise characteristics is a fundamental prerequisite before applying any signal analysis or modeling technique.

Noise contamination significantly affects both visual interpretation and automated ECG analysis systems. It may lead to incorrect peak detection, misclassification of heartbeats, and unreliable clinical decisions. In data-driven approaches, noise increases data dimensionality and reduces the signal-to-noise ratio, complicating model training and feature extraction.

Therefore, prior to advanced mathematical processing, ECG data must be analyzed with respect to its noise characteristics. This motivates the application of structured mathematical frameworks to separate meaningful cardiac information from undesired disturbances.

C. Singular Value Decomposition (SVD)

Singular Value Decomposition (SVD) is a fundamental matrix factorization technique in linear algebra that decomposes a real or complex matrix into three constituent matrices with well-defined geometric and algebraic interpretations. For any matrix $A \epsilon R^{m \times n}$, SVD factorizes the matrix as:

$$A = U \, \Sigma \, V^T$$

where

U : an m x m orthogonal matrix of left singular vectors

Σ : a diagonal matrix of singular values in decreasing order

V : an n x n orthogonal matrix of right singular vectors

The singular values represent the intrinsic energy or importance of each corresponding singular vector pair, providing a ranked decomposition of the matrix structure.

SVD is capable to give an optimal low-rank approximation. By retaining only the largest k singular values and their corresponding singular vectors, a matrix can be approximated as:

$$A_k = \sum_{i=1}^{k} \sigma_i u_i v_i^T$$

This truncated SVD minimizes the Frobenius norm of the reconstruction error among all rank-k approximations, making it mathematically optimal. This property has been widely exploited in data compression, dimensionality reduction, and signal representation.

SVD has been extensively applied in data analysis to reduce dimensionality while preserving dominant information, separate structured components from unstructured variations, or identify correlated patterns across observations.

D. Denoising Using SVD

Noise reduction is a fundamental problem in signal and data processing, particularly when dealing with real-world measurements that are often contaminated by random disturbances. One effective and widely studied approach for denoising is based on Singular Value Decomposition (SVD), which exploits the intrinsic low-rank structure of meaningful data.

The core idea behind SVD-based denoising is that useful signal components tend to be concentrated in the dominant singular values, while noise is distributed across smaller singular values. By decomposing a data matrix into its singular components, it becomes possible to separate structured information from unstructured noise.

In SVD-based denoising, a truncated SVD is applied by retaining only the largest k singular values:

$$A_k = U_k \Sigma_k V_k^T$$

This truncated reconstruction minimizes the magnitude of the approximation error, making it an optimal low-rank approximation in the least-squares sense. This property ensures that SVD-based denoising preserves the most significant structural information while suppressing noise-dominated components.

One of the key advantages of SVD-based denoising is that it does not require prior assumptions about noise distribution. Unlike frequency-domain filtering or model-based approaches, SVD adapts directly to the data structure, making it applicable across diverse domains such as image processing, time-series analysis, and pattern recognition.

In time-series applications, data are often embedded into structured matrices before applying SVD. This transformation allows temporal correlations to be captured as spatial structures in matrix form, further enhancing the separation between signal and noise. Studies have shown that dominant singular vectors correspond to coherent patterns, while noise manifests as high-rank, low-energy components.

III. IMPLEMENTATION

A. ECG Dataset

The implementation presented in this paper utilizes datasets from the MIT-BIH Arrhythmia Database (MITDB) and the MIT-BIH Noise Stress Test Database (NSTDB), which are publicly available through PhysioNet. These datasets provide ECG signals that are either clean or contaminated with realistic noise, making them suitable for evaluating ECG signal denoising algorithms.

Clean ECG signals are obtained from MITDB, while noise signals are taken from NSTDB, specifically Baseline Wander (BW) noise, which is characterized by slow and significant shifts in the ECG baseline. The noise is added to the clean ECG signal to generate a noisy ECG signal that realistically represents practical ECG recordings.

Sampling rate	: 360 Hz
Signal duration	: first 10 seconds of the recording
Noise type	: Baseline Wander (BW)
Axis unit	: seconds
Amplitude unit	: normalized

Prior to the denoising process, preprocessing is applied to the ECG signal by removing the mean value and normalizing the amplitude. These steps are performed to improve numerical stability during matrix decomposition.

B. Data Acquisition and Preprocessing

Data acquisition is performed using the wfdb library in Python, which enables direct access to PhysioNet records without the need to manually download files. The ECG signal is read using the wfdb.rdsamp function, which returns the ECG samples as a numerical array along with metadata such as the sampling rate.

```
sig, fields = wfdb.rdsamp(record,
                pn_dir="nstdb")
fs = float(fields["fs"])
```

To reduce computational complexity and improve processing efficiency, the signal is truncated to the first 10 seconds of the recording. This truncation does not affect the essential characteristics of either the ECG waveform or the baseline wander noise. The output of this stage is a one-dimensional discrete signal $x[n]$ with finite length.

```
if seconds is not None:
    N = int(seconds * fs)
    N = min(N, len(x))
```

ECG signals commonly contain a DC (direct current) offset, which is a constant component that does not carry physiological information. The presence of this offset can adversely affect matrix decomposition, as Singular Value Decomposition (SVD) is sensitive to scale and signal shifts. Therefore, the DC component is removed by subtracting the mean value from the signal. This step ensures that SVD focuses on signal variations rather than constant offsets.

```
x = x - np.mean(x)
```

To further improve numerical stability during the SVD process, the signal amplitude is normalized using its standard deviation. This normalization prevents dominance of large singular values due solely to scale differences and ensures that energy separation by SVD reflects the intrinsic signal structure rather than amplitude magnitude.

```
std = np.std(x)
    if std > 1e-12:
        x = x / std
```

C. Sliding-Window Mtarix Construction

SVD operates on two-dimensional matrices, whereas ECG signals are originally represented as one-dimensional arrays. Consequently, the ECG signal must be transformed into a matrix representation prior to decomposition. In this work, a sliding-window embedding method is used to construct the matrix, allowing repetitive patterns in the signal to be captured as dominant matrix structures.

Given a discrete signal:

$$x[n], \quad n = 0,1,2,\dots,N-1,$$

A window length L is selected such that

$$1 \le L \le N,$$

the number of columns is then defined as

$$K = N - L + 1.$$

A matrix $A \in R^{L\times K}$ is constructed as

$$A[:,j] = \begin{bmatrix} x[j] \\ x[j+1] \\ \vdots \\ x[j+L-1] \end{bmatrix}, \quad j = 0,1,\dots,K-1.$$

Each column of the matrix represents a segment of the ECG signal of length L, shifted by one sample relative to the previous column.

```
def build_sliding_matrix(x:np.ndarray, L:int)
→ np.ndarray:
    N = len(x)
    if not (1 <= L <= N):
        raise ValueError("1 <= L <= len(x)")
    K = N - L + 1
    return np.column_stack([x[j:j + L] for j
in range(K)])
```

The choice of window length L significantly affects the SVD results and denoising quality. A window that is too small provides limited contextual information, reducing the effectiveness of SVD, while a window that is too large increases computational cost and may incorporate irrelevant patterns. In this implementation, a window length of L = 100 samples is used, which corresponds to approximately 0.28 seconds at a sampling rate of 360 Hz. This duration sufficiently captures key features of a single ECG cycle while preserving temporal resolution.

D. Singular Value Decomposition (SVD)

SVD is performed using the numpy.linalg.svd function with the full_matrices=False option to improve computational efficiency. This configuration produces a reduced decomposition that matches the effective dimensions of matrix A, thereby reducing computation time without sacrificing essential information.

```
U, s, VT = np.linalg.svd(A, full_matrices=False)
```

E. Rank-r Approximation Denoising

In SVD-based denoising, components associated with large singular values represent the dominant signal structure, while components corresponding to small singular values are typically associated with noise or random fluctuations. Therefore, denoising is achieved by retaining only the dominant components and discarding the remaining ones. This approach is known as rank r approximation or truncated SVD, and is expressed as

$$A_r = U_r \Sigma_r V_r^T \ ,$$

where

$U_r \ contains \ the \ first \ r \ columns \ of \ U$
$\Sigma_r \ is \ a \ diagonal \ matrix \ containing \ \sigma_1, .., \sigma_r$
$V_r^T \ contains \ the \ first \ r \ rows \ of \ V^T$

The choice of r directly affects denoising performance. If r is too small, important ECG information may be lost, and if r is too large, noise may remain in the reconstructed signal. In this implementation, the value of r is selected based on the cumulative energy of the singular values, defined as

$$E_{total} = \sum_{i=1}^{L} \sigma_i^2$$

The smallest value of r is chosen such that

$$\frac{\sum_{i=1}^{r} \sigma_i^2}{\sum_{i=1}^{L} \sigma_i^2} \geq \eta$$

where η denotes the retained energy fraction. This criterion ensures that most of the signal energy is preserved while components associated with noise are suppressed.

```
def choose_r_by_energy(s: np.ndarray,
e_keep: float = 0.95) → int:
    e = s**2
    cum = np.cumsum(e) / np.sum(e)
    r = int(np.searchsorted(cum, e_keep)+1)
    return max(1, min(r, len(s)))
```

```
r = choose_r_by_energy(s, e_keep=e_keep)
A_r = U[:, :r] @ np.diag(s[:r]) @ VT[:r, :]
```

F. Reconstruction to a 1D Signal

After obtaining the denoised matrix A_r, it must be converted back into a one dimensional signal $\hat{x}[n]$ for visualization and analysis. Since the sliding-window matrix is constructed according to

$$A[i,j] = x[i+j] \ ,$$

A single sample $x[n]$ appears in multiple matrix entries (i,j) that satisfies i+j = n. Therefore, diagonal averaging is employed to reconstruct the signal by averaging all elements along the same anti-diagonal.

$$\hat{x}[n] = \frac{1}{|\Omega_n|} \sum_{(i,j)\epsilon\Omega_n} A_r[i,j]$$

$$\text{where } \Omega_n = \{(i,j) | i+j = n\}$$

```
def diagonal_averaging(A: np.ndarray) →
np.ndarray:
    L, K = A.shape
    N = L + K - 1
    out = np.zeros(N, dtype=float)
    cnt = np.zeros(N, dtype=int)

    for i in range(L):
        for j in range(K):
            out[i + j] += A[i, j]
            cnt[i + j] += 1

    out /= cnt
    return out
```

IV. RESULTS

The original noisy ECG recording is visualized in the following graph.

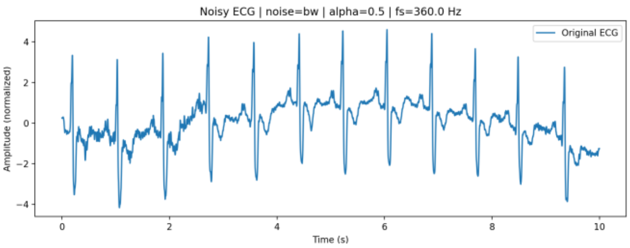

Figure 1. Original ECG graph
Source: Author

After applying the denoising procedure, the denoised ECG signal is shown below.

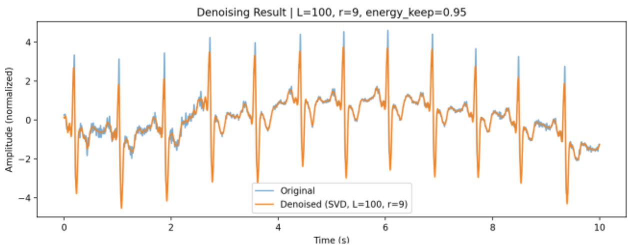

Figure 2. Original and Denoised ECG graph Comparison
Source: Author

Based on the visualization results, a noticeable reduction in baseline wander can be observed. In the noisy ECG signal, the baseline exhibits slow drifting behavior, which is characteristic of baseline wander noise typically caused by respiration, electrode motion, or patient movement. After denoising, this baseline drift is significantly reduced, resulting in a more stable baseline centered around zero. This improvement enhances the overall readability of the ECG signal and facilitates more reliable interpretation of cardiac activity.

Furthermore, the denoising process successfully preserves ECG morphology. Critical components of the ECG waveform, particularly the QRS complex (the initial downward deflection, the sharp upward spike, and the subsequent downward deflection) remain clearly visible after denoising. Other waveform components, such as the P and T waves, are also largely preserved. No significant phase shift, peak clipping, or waveform distortion is observed, indicating that the denoising process does not compromise essential physiological information. This preservation is crucial, as accurate morphology is required for clinical analysis such as heart rate estimation and arrhythmia detection.

The selected rank value $r = 9$ is obtained based on a cumulative energy threshold of 95%. This result indicates that the majority of the signal energy is captured by nine dominant SVD components. By retaining these components, the structured ECG signal is preserved while noise components associated with smaller singular values are effectively reduced. This demonstrates that energy-based rank selection provides a practical and data-driven balance between noise reduction and signal preservation, eliminating the need for manual trial-and-error selection of the rank parameter.

Despite the promising denoising performance, several limitations of the SVD-based method are observed. The effectiveness of denoising strongly depends on the choice of window length L and rank r. An inappropriate selection of these parameters may result in either insufficient noise removal or excessive signal distortion. Consequently, there is no universally optimal value of r, and parameter tuning must be performed carefully based on signal characteristics and noise type. This dependence can be a limitation in automated or real-time applications, where adaptive parameter selection may be required.

In some non-QRS segments, the reconstructed signal appears smoother than the original. This phenomenon indicates over-smoothing, which occurs when low-energy components containing subtle signal details are removed along with noise. Although this effect is relatively minor in the present implementation and does not significantly affect major diagnostic features, it highlights that SVD is not perfectly selective in separating noise from low-energy but potentially meaningful signal components.

Additionally, the implementation in this paper focuses on baseline wander noise, which is globally structured, low-frequency, and smoothly varying. These characteristics align well with the low-rank approximation assumption of SVD, enabling effective noise reduction. However, for other types of noise such as electromyographic (EMG) noise, which is more irregular and distributed across many singular values, SVD-based denoising may be less effective. In such cases, truncating singular values risks removing important ECG components along with noise, suggesting that hybrid approaches or complementary filtering techniques may be more suitable.

V. Conclusion

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

VI. Appendix

Github: https://github.com/gabriellaalubis/SVD-Denoising-for-ECG

VII. Acknowledgment

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PERNYATAAN

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.

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